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Communication regimes in opinion dynamics: Changing the number of communicating agents

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Abstract. This article contributes in four ways to the research on time-discrete continuous opinion dynamics with compromising agents. First, communication regimes are introduced as an elementary concept of opinion dynamic models. Second, we develop a model that covers two major models of continuous opinion dynamics, i.e. the basic model of Deffuant and Weisbuch as well as the model of Krause and Hegselmann. To combine these models, which handle different numbers of communicating agents, we convert the convergence parameter of Deffuant and Weisbuch into a parameter called self-support. Third, we present simulation results that shed light on how the number of communicating agents but also how the self-support affect opinion dynamics. The fourth contribution is a theoretically driven criterion when to stop a simulation and how to extrapolate to infinite many steps.

1 Introduction

Opinion dynamic (OD) models describe the process of opinion formation in groups of individuals. We focus on continuous opinion dynamics (COD) with compromising agents in a time-discrete world. This implies that an opinion is a continuous value between zero and one. In every time step, each agent adapts his opinion towards the opinions of a set of perceived agents, while the new opinion is between minimum and maximum of the own and all perceived opinions (compromising). A common feature among many models of continuous opinion dynamics is bounded confidence, which describes the fact that opinions far away from an agent's own opinion do not exhibit any influence on this agent.

Two models of continuous opinion dynamics have received much attention. One the one hand, the model of Deffuant and Weisbuch (DW model) [3, 4, 14, 15] and on the other hand the model of Krause and Hegselmann (KH model) [9]. A fundamental difference between these two models is the number of agents that communicate. In each time step in the DW model two randomly chosen agents mutually perceive their opinions, while in the KH model all agents perceive all other agents. The same tendency towards extreme models regarding the number of communicating agents can be found in the related literature on discrete opinion dynamics (see for instance [12] but also [5]).

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The difference between the DW model and the KH model is in a dimension that Hegselmann calls the update mechanism [8]. We would label it the communication regime, which relates the dimension more to social reality than to models. Social reality may restrict the process of communication between agents such that communication between all agents at the same time (as done in the KH model) but also communication between two randomly chosen agents (as done by the DW model) appear as extreme cases. Apart of restricting the number of communicating agents we might think of social or physical networks bounding the set of agents who could potentially affect an agent. There are some simulation results about opinion dynamics on social networks, see for instance [1, 5, 13]. But static social networks are not the only restriction we may think of. It could make a difference if we first communicate with our mother and then with our friends or vice versa. Hence, also temporal aspects can play a major role. In other words, we want to introduce rules about who is communicating with whom at what time. We call such rules and restrictions the *communication regime* of an opinion dynamics model. We call it 'regime' because it is independent of the opinions the agents have; it is a parameter of the model. The communication regime includes the underlying social network but also how this may change over time. In a mathematical sense the communication regime is a (temporal) sequence of networks of who perceives whose opinion. It can be treated as module of OD models.

Another module of opinion dynamic models is what we call the *mental model*. The mental model introduces rules how agents adapt their opinions based on a set of other agents' opinions. It models perceptual or information processing biases, e.g. bounded confidence. By many parallel and sequential communications these biases are multiplied in a group such that new sometimes even more complex group dynamics can be observed. Increasingly the mental model is treated as a module of OD models independently of the communication regimes. For instance the mental model of the basic DW model is put on network structures (see [1, 13]), or the KH model is developed with a mental model that captures discrete opinions instead of continuous opinions (see [5]).

In the section 2 we provide a general model of continuous opinion dynamics that extends the general model presented in [9] by explicitly including communication regimes. In section 3 we provide examples how communication regimes can be used to capture parts of other opinion dynamic models and how they can be used to model different kinds of social networks. The major difference between the basic model of Deffuant and Weisbuch on the one hand and the model of Krause and Hegselmann on the other hand is the communication regime. Hence, our general model should be able to provide a model that contains both models. Such a model is provided in the section 4. As a prerequisite we unify the mental models; thereby we re-define the convergence parameter μ (now called self-support) of the DW model and introduce this idea into the KH model. Section 5 summarizes simulations that show how the differences in the communication regime influences the dynamics. We also show that the impact of self-support is moderated by the communication regime.

2 A Generalized Model of Continuous Opinion Dynamics with Communication Regimes

In this section we develop a generalized COD model. Its mental model is based on weighted averaging. Communication regimes are implemented by sequences of matrices. We formulate the model as a time-discrete dynamical system written in 'matrix language', where the agents are associated with the indices of the matrix. As we will see later this formalization of communication regimes also supports the analysis of social networks in opinion dynamics. Our general model is a modification of the general model that is presented in [9].

Agents and their opinions Consider a set $\underline{n} := \{1, \ldots, n\}$ of agents. We call the vector $X(t) \in \mathbb{R}^n$ an *opinion profile.* $X_i(t)$ for $t \in \mathbb{N}_0$ represents the opinion of agent i at time t. The initial opinion profile is given by $X(0)$.

Definition 1 (communication regime). *A sequence $C(t) \in \{0,1\}^{n \times n}$ for $t \in \mathbb{N}$ is called a* communication regime *if the diagonal is positive in every matrix.*

A communication regime is thus a sequence of $(0,1)$-matrices, which might also be called communication matrices. If $c_{ij}(t) = 1$ then agent i perceives agent j at time step t. Hence, $C(t)$ is the adjacency matrix of the network of who perceives whom at time step t, i.e. the i-th row in $C(t)$ marks the *set of perceived agents* of agent i. The positive diagonal means that self-communication should always be possible. For the set of perceived agents we define the mental model as an abstract weighted averaging rule. For a discussion and analysis of different abstract averaging rules in opinion dynamics see [10].

Definition 2 (averaging rule). *Let $C(t)$ be a communication regime and $i \in \underline{n}$ be an agent. A function* $\mathrm{weight}_i : \underline{n} \times \mathbb{R}^n \times \mathbb{N} \to [0,1]$ *is called an* averaging rule *if it holds*

$$\begin{aligned} \forall t \in \mathbb{N} \quad \forall X \in \mathbb{R}^n \quad &: \quad \sum_{j=1}^{n} \mathrm{weight}_i(j, X(t), t) = 1 \\ \forall t \in \mathbb{N} \quad \forall X \in \mathbb{R}^n \quad \forall j \in \underline{n} : &\quad c_{ij}(t) = 0 \Longrightarrow \mathrm{weight}_i(j, X(t), t) = 0 \end{aligned}$$

An averaging rule $\mathrm{weight}_i(j, X(t), t)$ shows how much weight agent i puts on opinion of agent j at time step t. The first constraint implements a normalization of the weights an agent puts on all agents (including himself), while the last constraint ensures that agent i can only put weights on actually perceived agents. The weight function can implement many different approaches. Based on the weight function we can complete the construction of the mental model. Thereby we adopt the idea of the confidence matrix as it is also done by [9, 11]; but additionally we require this matrix to be along with the communication regime. Finally we can recursively define the process of opinion formation.

Definition 3 (confidence matrix). *Let $C(t)$ be a communication regime, $X(t)$ an opinion profile and let* weight_i *be an averaging rule for all $i \in \underline{n}$, then*

$$A(t, X(t), C(t))_{[i,j]} := \text{weight}_i(j, X(t), t)$$

is called a confidence matrix.

Due to definition 2 for every time step t matrix $A(t, X(t), C(t))$, the communication matrix, is row-stochastic. Further on, it holds that $A(t, X(t), C(t)) \leq C(t)$. This represents, that the mental model is restricted by the communication regime. Finally we define the process.

Definition 4 (COD process). *Let $C(t)$ be a communication regime, $X(0) \in \mathbb{R}^n$ be an initial opinion profile and* $\text{weight}_1, \dots, \text{weight}_n$ *averaging rules for all agents in $\underline{n}$. The* process of continuous opinion dynamics *is a sequence of opinion profiles $(X(t))_{t \geq 0}$ recursively defined through*

$$X(t+1) = A(t, X(t), C(t))X(t).$$

3 Examples for Communication Regimes

Our general model explicitly introduces the concept of communication regimes. Some examples show the variety of structures that may be modeled.

Communication in groups If C is the unit matrix then there is no communication at all. If contrary C contains only ones then it is the communication regime as modeled in [9], i.e. every agent perceives every other agent. We can see this communication regime as a commission trying to pool the commission members' opinions where every agent gets the opinion of every other agent before starting to rethink about his own new opinion. Between 'no communication' and 'communication with all agents' we can define communication in groups of m agents, where all agents in a group perceive only the opinions of their group members. We may think of people going to lunch each day and discuss with their neighbors. The communication regime of the DW model is of this type with a group size of two. We may think of it as a society where in every step one agent spontaneously decides to phone another agent out of his phone book to try to compromise with. We summarize all these communication regimes as *m-communication regimes*, where parameter m characterizes the group size. For binary opinions such a parameter was investigated in a model by [7].

Definition 5 (m-communication regime). *Let $\underline{n}$ be a set of agents and $m \leq n$. We call a communication regime $C(t)$* m-communication regime *if for all $t \in \mathbb{N}$ it holds that $C(t)$ is symmetric and there are exactly m rows containing exactly m ones and the rest zeros. All other rows contain only zeros, except the diagonal entries, which are ones.*

For a system with n agents there is only one n-communication regime, but there may be many different 2-communication regimes. Models with m-communication regimes with m less than n frequently include a random choice of the actually applied communication regime. For each matrix from the communication regime m agents are randomly selected to communicate. We call these classes of communication regimes *random m-communication regimes*. The random n-communication regimes is a special case not including any random choice. In the DW model the communication regime is a random 2-communication regime; thus for each step of the opinion dynamics there is a random pairwise communication. Compared to the KH model this adds an additional source of randomness and perhaps additional variance to the model. The DW model and the KH model are extreme cases of this class of communication regimes, i.e. for $m = 2$ and $m = n$, respectively. By formalizing random m-communication regimes we implement an intuition that is also mentioned in [8].

Social networks If we only apply $C(t) = C$ then we get the concept of networks as applied by French [6], with C being the adjacency matrix of the social network. Matrix C formalizes the structure of a static social network. If we are less restrictive and only require $C(t) \leq C$ for all t then we define the *underlying social network* as the smallest C that satisfies $C(t) \leq C$. If communication partners are randomly chosen according to a social network represented by a matrix C then the underlying social network is C. Such random communication on a network C is implemented for instance by [13].

Spatial or temporal structures Another very specific example is $c_{ij} = 1$ for all $i, j \in \underline{n}$ with $|i - j| \leq 1\}$; this is "communication in a line", where line refers to a spatial concept. Many more complex static communication regimes as for instance the Moore-neighborhood or other spatial arrangements fit into this concept of communication regimes.

Since $C(t)$ depends on t the structure who perceives whom may change over time and definitions of time schedules and frequencies are possible, e.g. close neighbors were perceived more frequently than agents of greater distance. This also allows for the definition of multi-layer social networks resulting in different communication frequencies as it can be found for instance in [2].

4 The Smallest Model covering DW and KH

In this section we present a general model that explicitly addresses the modeling of communication regimes. In the previous section we saw that the DW model as well as the KH model apply random m-communication regimes, while they differ only in the parameter m. In this section we want to utilize this for a unification of these two influential models. To find a parameterized model covering both, DW model (with globally uniform uncertainty and without relative agreement) and KH model, we have to make the communication regimes and the mental models compatible; this means we have to define an appropriate communication regime and an averaging rule.

Since both models apply random m-communication regimes, a general model containing both specific model therefore only needs to incorporate such communication regimes with an integer parameter m less or equal to the number of agents n. Thus we restrict us in this particular model to random m-communication regimes, but we must keep in mind, that for $m = n$ there is no random choice in the communication regime.

The two mental models are very similar in the way that they both use a threshold of *bounded confidence* respectively *uncertainty*. We will call it ε according to the KH model. The second parameter we use has its origins in the DW model. We call it *self-support* μ. This parameter allows formulating an averaging rule that is able to reproduce both models. Since we specify a sub model of our general model the following definition fulfills the properties required by 2.

Definition 6 (averaging rule with ε and μ). *Let $C(t) \in \{0,1\}^n$ be a communication regime and $i \in \underline{n}$ be an agent, $X(t)$ be an opinion profile and $0 < \varepsilon, \mu \in [0,1]$. Then we define the* averaging rule *as*

$$\text{weight}_i(j, X) = \begin{cases} \mu + \frac{1-\mu}{|I(i,X)|} & \text{if } j = i \\ \frac{1-\mu}{|I(i,X)|} & \text{if } j \in I(i,X) \text{ and } j \neq i \\ 0 & \text{otherwise} \end{cases}$$

$$\text{with } I(i,X) := \{j \in \underline{n}, \mid c_{ij} = 1 \text{ and } |X_i - X_j| \leq \varepsilon\}$$

(We omitted the specific time step for abbreviation.)

Parameter μ is not the same as the parameter μ in the DW model. To distinguish them we label the original parameter of the DW model μ_D. If we set $\mu = 1 - 2\mu_D$ and $m = 2$, then our model equals the DW model. The assumption of $0 < \mu_D \leq 0.5$ that is done in DW model corresponds to $0 \leq \mu < 1$ in our model. The parameter μ_D implements a tendency how much another opinion influences the own opinion. Since our model implements a multilateral communication this approach was not easily extended. Should we assign a fixed cumulated influence to all other influential opinions? We rejected this idea because we aimed at a combination of the DW model and the KH model, and the latter is incompatible with this idea. Instead we considered a lower limit of the weight assigned to the own opinion. We call this the self-support. On top of this value we add a value that depends on the number of influential opinions. This allows a smooth transition from the DW mental model to the KH mental model, for which we have to set $\mu = 0$.

All together we have a unified basic model of continuous opinion dynamics that allows for the analysis of the impact of different numbers of communicating agents. It includes a special class of communication regimes that is independent of social networks or in other words a communication regime that has only the fully connected network as its underlying social network. It incorporates two important basic models that are applied in many articles on continuous opinion dynamics with compromising agents, i.e. the model of Deffuant and Weisbuch and the model of Krause and Hegselmann. Actually both extremes happen in

real social structures. But also meetings with $m = 3$ or 4 or 10 or 50 people occur. Independent variables of our model are the number of agents n, the number of communicating agents m, self-support μ and the bounded confidence parameter ε. Actually the initial profile $X(0)$ and the randomly chosen m-communication regime $C(t)$ are also free variables of the model, but we will treat them as endogenous random choices being equally distributed within the defined bounds. Due to this random effect we are forced to run many simulations with different randomly chosen $X(0)$ and $C(t)$.

5 Simulations

For exploring the model presented in the last section we first define three dependent variables: (1) final number of clusters, (2) convergence time, and (3) last split time. Doing this, we show that we apply a simulation mechanism that is equivalent to simulations of infinite many time steps. We then report simulation results that explore the effect of changing the number of communicating agents. Since our model also contains the parameter self-support we additionally run simulations that shed light on this parameter and – more important – how it interacts with the number of communicating agents.

5.1 Clusters, convergence and last split

The most important outcome of an opinion dynamics process is the final number of clusters. In many papers on opinion dynamics, a cluster usually refers to a set of agents that hold the same opinion. For continuous opinion dynamics this can be a difficulty because the equality might only hold as a long-term limit. To solve this problem we will define fully connected classes of agents and show that for states with only fully connected classes, we can calculate the long-term limit of the dynamics.

It can be shown that each process in our general model covering the DW and the KH model reaches a time step in which the agents split for all further time steps into the same disjoint classes of agents for which two conditions hold: (1) the distance of each agent in one class to each agent in another class is greater than ε and (2) for one class it holds that the maximal opinion in the class minus the minimal opinion in the class is less or equal ε. (For a proof see [11].) Thus for each class it holds that if some agents of this class get in contact due to the communication matrix then everybody trusts everybody directly. Thus we call this class fully connected. Further, for a fully connected class we can be sure that the opinions converge to the same opinion, so to a cluster, but this can last in some cases infinite time steps (due to μ and specific communication regimes).

For continuous opinion dynamics reaching a state where all classes are fully connected seems to be a convincing definition of convergence. But the positions of the evolving clusters are not unambiguously defined. Hence this definition of fully connected classes makes an analysis of the distribution of clusters difficult. To deal with this problem we apply the following lemma.

Lemma 1. *Let $X \in \mathbb{R}^n$ and let $A \in \mathbb{R}^{n \times n}$ be row-stochastic and symmetric. Let $\bar{X}$ be the arithmetic mean of X, i.e. $\bar{X} := \frac{1}{n}\sum_{i=1}^{n} X_i$. It holds $\bar{AX} = \bar{X}$.*

Proof.

$$\bar{AX} = \frac{1}{n}\sum_{i=1}^{n}\sum_{j=1}^{n} a_{ij}X_j = \frac{1}{n}\sum_{j=1}^{n}(X_j \underbrace{\sum_{i=1}^{n} a_{ji}}_{=1}) = \frac{1}{n}\sum_{j=1}^{n} X_j = \bar{X} \qquad \square$$

Due to fully connectedness and definition 6 we know for our model that for a system with only fully connected classes the confidence matrix A is always row-stochastic and symmetric. The lemma states that the average of all opinions of the fully connected class always stays the same, thus even in the limit case. This allows to calculate the long-term limit of the dynamics.

All together we have a precise criterion when to stop a simulation instead of applying a heuristic. Our criterion stops at the point in time when we have only fully connected classes. The time needed to reach this state is the *convergence time*. We then calculate the long-term limit of the final *number of clusters*. Beside the number of clusters and convergence time we also look at the *last split time*. This is the point in time when the final set of classes emerges, but these classes have not necessarily converged into fully connected classes, i.e. only the first condition for fully connected classes holds. In simulations the last split time can only be determined after determining the convergence time, because for not fully connected classes we cannot easily determine whether it will split again or not. Despite last split time and convergence time seem to be very similar, we will see in simulations that they can show different behaviors.

5.2 Changing the number of communicating agents m

For exploring the effect of changing the number of communicating agents, m, we keep parameter μ constant at zero. For $m = n$ the model reduces to the KH model and for $m = 2$ we get the DW model with $\mu_D = 0.5$. The effect of changing parameter μ is explored later in this paper.

To get a first idea, how m may influence the dynamics we run nine simulations. Figure 1 shows the nine processes of opinion dynamics, $\varepsilon \in \{0.1, 0.2, 0.3\}$, $m \in \{2, 20, 100\}$), and all with the same initial profile of 100 randomly chosen opinions. In each graphic, the x-axis is time and the y-axis is the opinion of every agent. For low ε the agents form several clusters. For higher ε the number of clusters decreases. With even higher ε the agents find consensus. All models show the same behavior but it seems that the same ε causes slightly more clusters in the DW model. Figure 1 shows and this also holds if the time needed to converge is transformed such that in every step the same number of agents adapt their opinions (e.g. dividing t by $\frac{100}{m}$ for the DW model) that for smaller values of m it takes longer to converge. Similar graphs and the same observations are reported in [8].

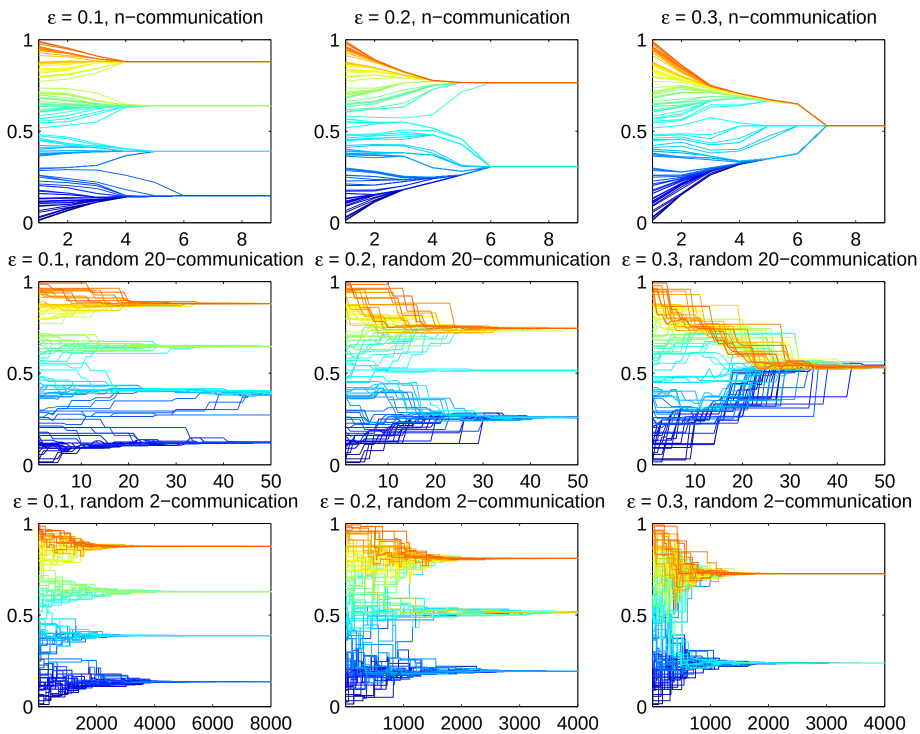

Fig. 1. Examples: 100 agents with randomly chosen initial opinions for ε from $\{0.1, 0.2, 0.3\}$ and for $m = n$ (KH model), $m = 20$ and $m = 2$ (DW model)

To make the dynamics more visible we take a look at a very simple example with three agents. Thereby we assume that for different numbers of agents the fundamental micro behaviors will not change significantly. Consider three agents with opinions 0, 0.5, and 1, all with $\varepsilon = 0.5$ and any self-support $\mu < 1$, e.g. $\mu = 0.4$. For the DW model two clusters will emerge, either with opinions 0.0 and 0.75 or with 0.25 and 1.0 (depending on the first communications). If the extreme agents communicate they ignore each other. If the middle agent communicates with an extreme agent it will adapt to the extreme and leave the space where the other extreme agent could influence it. For the KH model one cluster emerges with opinion 0.5. If we further increase epsilon above 0.5 then the probability to reach consensus increases for the DW model until finally the probability reaches 1. Between the extremes the particular patterns of communication influence the probability of convergence to one cluster.

All together we can state our first hypothesis: *The higher the number of communicating agents, i.e. the bigger m, the less the number of expected clusters and the less the convergence time and last split time.*

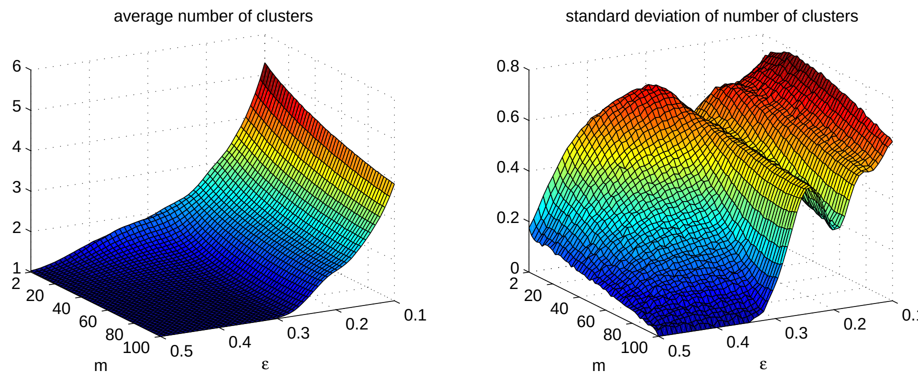

Fig. 2. Simulation results: epsilon and number of communicating agents versus average number of clusters (left) and standard deviation of the number of clusters (right)

To check our hypothesis we run simulations with 100 agents for different scenarios with 50 stages for m as well as 50 stages for ε. Every single scenario is simulated 10.000 times with randomly selected initial opinion profiles. Figures 2, 4, and 3 visualize the results.

On average for many different opinion profiles the number of clusters increases the fewer agents communicate. In other words, for fewer communicating agents we need more confidence, i.e. more open-minded agents, to reach the same number of clusters. The standard deviation of the number of clusters in many cases decreases for bigger m; but for specific ranges of ε the standard deviation decreases with an increase of m for very small m (below 25) and decreases for bigger m; it has a maximum bigger than 2 and less than n.

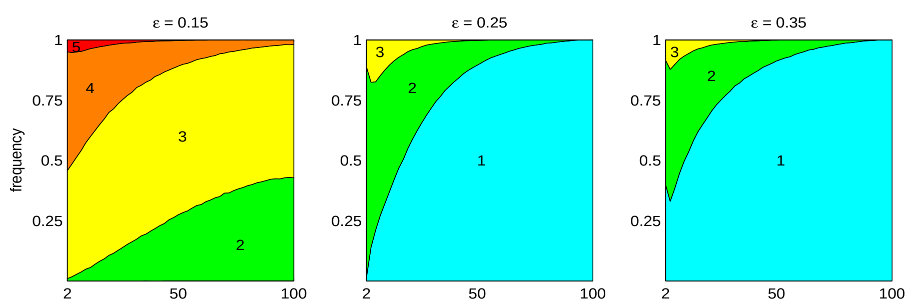

Fig. 3. Simulation results: Number of communicating agents versus probability of particular numbers of clusters for three instances of epsilon, i.e. $\varepsilon = 0.15, 0.25, 0.35$

Figure 3 illustrates these findings more detailed. For three different ε, i.e. $\varepsilon \in \{0.15, 0.25, 0.35\}$, we plotted the probabilities of reaching a particular number of

clusters. The number of communicating agents has 30 stages and every setting was simulated 50.000 times with randomly chosen initial opinion profiles. For $\varepsilon = 0.35$ we observe that for an increasing m the average number of clusters increases for very small m, while for more than 3 agents it decreases as everywhere else. The same effect is not easy to see but can be found in Figure 2 for epsilon between 0.25 and 0.4. Despite this effect does not appear for $\varepsilon = 0.25$ and for the average number of clusters, a detailed inspection of 3 shows that also in this case there is a specific behaviour fro small m. The reason for this effect remains open for future research, but we can say that this effect gets smaller for bigger self-support μ (we omit the graphs here).

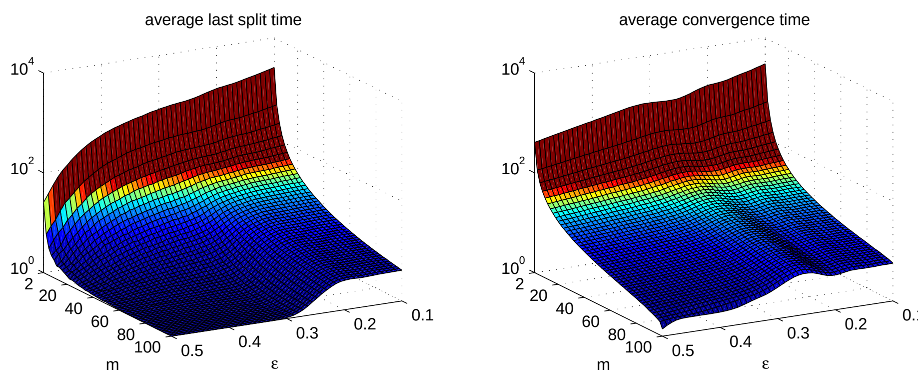

Fig. 4. Simulation results: epsilon and number of communicating agents versus average last split time (left) and average convergence time (right)

Figure 4 shows the average last split time and the average convergence time. It holds that the more agents communicate the less steps agents need to converge. The graphs show another very interesting behavior that holds for the DW model as well as for the KH model: Despite there is a general tendency that the convergence time decreases with an in increasing ε, there is a maximum between $0.2 < \varepsilon$ and $\varepsilon < 0.3$. This does not hold for the last split time. Because this effect is not specific for changing m we omit a further analysis, but we hypothize that in the interesting area there is a tendency to polarize temporarily, but convergence occurs in the long run. In such cases convergence time increases significantly since convergence of two polarized groups take much time.

5.3 Changing the self-support μ

For analyzing the effect of m we set μ to zero, because for this case both extremes, $m = 2$ and $m = n$, represent instances of the original DW model and KH model, respectively. If we change μ then for $m = 100$ we do not have the original KH model anymore. We end up with a model that includes self-support into the KH model. We now analyze how such changes affect the opinion dynamics.

Since μ is based on the convergence parameter of the DW model we can adapt an observation from [4] and hypothize that *the smaller the self-support, the less is the convergence time*; but, does it affect the average number of clusters?

Let us take a second look at the simple three-agent example introduced in subsection 5.2. Consider the example for the DW model and ε bigger than 0.5. The slower the middle agent moves, i.e. the higher its self-support, the lower is the probability that it "looses contact" to one of the extreme agents. And the lower this probability the higher is the probability that the middle agent forces the extreme agents to reach consensus. Hence we can hypothize that *there are cases where a higher self-support decreases the expected number of clusters*. This effect of changing μ could be strongest if extreme agents have to move a long way while the middle agents have much time to move away; hence, for big epsilon (where consensus is very likely) the effect could be strongest. But on the other hand, the slower the extreme agents move the higher is the probability that the middle agent looses contact to an extreme agent, because the extreme agents do not get closer to the middle agent quickly enough. The second effect could also have its strongest impact for cases where the extreme agents have to go a long way to reach a stable state, i.e. consensus. For such conditions we might expect cases where an increase of the self-support parameter increases the number of evolving clusters. Hence we can also hypothize that *there are cases where the number of clusters increases due to an increase in the self-support μ*. Both effect seem to have their strongest impact for big epsilon. Simulations will show how these contrary forces work.

To see that both effects are working see Figure 5, where we consider an example with a fixed initial opinion profile of 100 agents, $m = 100$ (remember that there is no randomness in the n-random communication regime, i.e. the KH model), $\varepsilon = 0.25$, and three stages for the parameter μ. We see that raising μ from 0.0 to 0.3 takes the number of clusters from 2 to 1, but raising μ further from 0.3 to 0.6 takes it back to 2 clusters.

Figure 6 presents how μ affects on average the final number of clusters in the DW model (left) and in the KH model (right), respectively. We run simulations

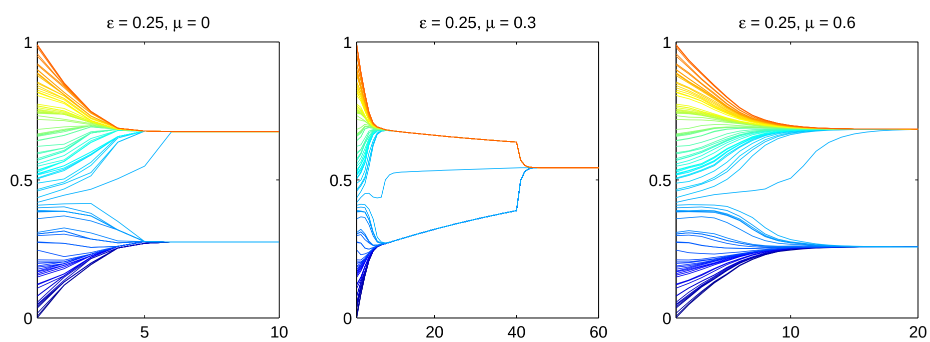

Fig. 5. Examples for one initial opinion profile $X(0) \in \mathbb{R}^{100}$, $\varepsilon = 0.25$ and $\mu = 0, 0.3, 0.6$

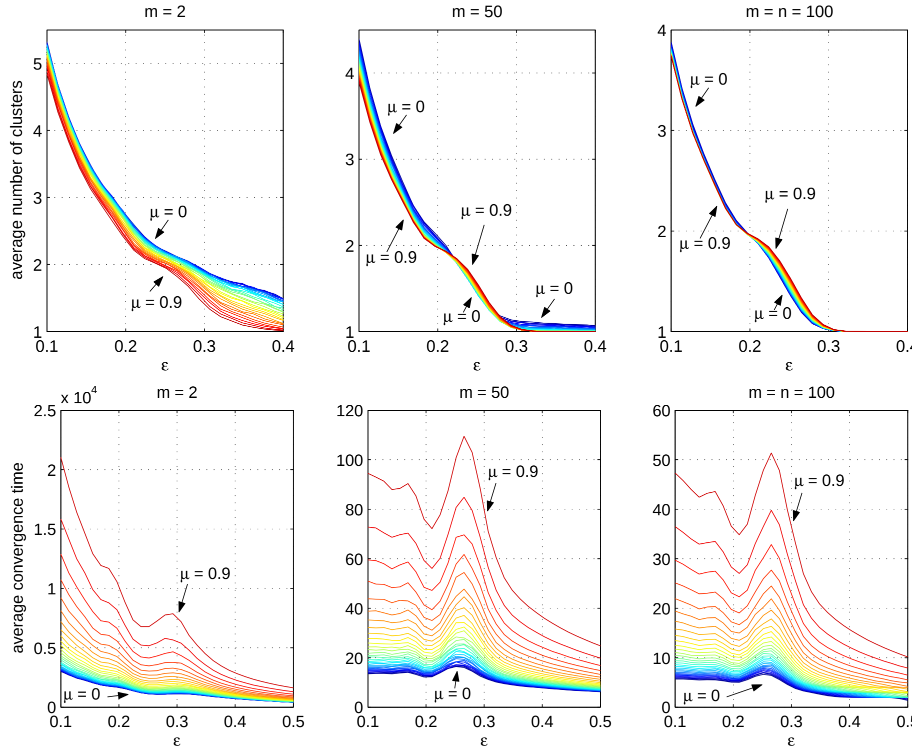

Fig. 6. Simulation results: epsilon versus average number of clusters for different μ and for three different m. red represents $\mu = 0.9$ and blue represents $\mu = 0.0$.

for 100 agents, three stages of m, 30 stages of ε, and 30 stages for μ, while every setting was simulated 5.000 times with random initial opinion profiles.

We can see that for the DW model ($m = 2$) an increase of μ decreases the number of clusters. For the KH model ($m = n = 100$) we recognize a different case. For small ε the direction of the effect is the same as for the DW model (but the effect is much smaller); but for big ε it is inverse (but still the effect is smaller than for $m = 2$). In fact, an increase in μ for big ε increases the average number of clusters.

Regarding the convergence time we can confirm our hypothesis and the observation reported in [4], i.e. the bigger μ, i.e. the smaller μ_D, the longer the agents need to converge.

However, our results regarding the number of clusters seem to be in partial contradiction with one observation mentioned in [4]: "μ and N [$=n$ in our model] only influence convergence time and the width of the distribution of final opinions (when a large number of different random samples are made)." The difference might be due to the fact that we do not exclude "wings", i.e. asymmetric peaks with a vertical bound of either 0 or 1, from our data set. Independent of this,

we see that the effect of μ changes when the communication regimes changes. Hence, it might be interesting how the opinion dynamics change if networks are introduced as part of communication regimes.

Until now we have only analyzed averages of many different initial opinion profiles and discussed the major effects. In this paragraph we briefly summarize observations that are not dominant or that appear only for specific or fixed initial opinion profiles. Due to page limitations we do not explain these effects in detail. For changing the number of communicating agents m for given epsilon and given self-support we observed two minor effects: For some epsilon between 0.28 und 0.4 a change from $m = 2$ to $m = 3$ or $m = 4$ increases the number of clusters, while usually it decreases. Also, for some initial opinion profiles the change from $m = n - 1$ to $m = n$ causes some non-monotony, because by this change we eliminate randomness in the communication regime. For changes in epsilon for $m = n$ and $\mu = 0$ (KH model) non-monotonic behavior is already reported [11]; however, from our simulations we can report this also for the DW model for specific initial opinion profiles (e.g. equidistant initial opinions). Figure 5 shows that non-monotonic behavior also appears for changes in μ for specific initial opinion profiles. All together, we can summarize that the average behavior of our model seems to be relatively smooth and clean, but for specific parameter settings and specific initial opinion profiles there are qualitative deviations from the average behavior.

6 Conclusion and Outlook

This article contributed to the literature on continuous opinion dynamics in four ways. First, we provided a general model of continuous opinion dynamics that distinguishes the mental model from communication regimes. We discussed how ideas like social networks and existing models are related to the idea of communication regimes. Second, based on the general model we developed a more specific model that contains two of the most influential models of continuous opinion dynamics. The crucial parameter that distinguishes these two models is the number of agents that communicate in one step, i.e. the communication regime. The unifying model allows an exploration of the space between the extremes. We were able to show that an increase in the number of communicating agents mostly leads to a decrease of the average number of emerging clusters. There is a small difference in the mental models of the Deffuant and Weisbuch model and the Krause and Hegselmann model. The first introduces a parameter that we picked up and that we have implemented similar as self-support. We were able to show that there is an interaction effect between the number of communicating agents the self-support. A fourth contribution is not related to model analysis but to the implementation of simulations. We pointed out that for the unifying model, which covers the DW model as well as the KH model, there is a theoretically driven condition that allows to stop a simulation and to calculate the simulation result for a simulation with infinite many steps. Having this, we are able to "simulate" infinite many steps.

Several question remain open for future work. We need to explore the effect of the number n of agents in the system and the interaction effects between all parameters m, n, μ, and ε. We also have to explore heterogeneous societies. For implementations we want to extend our stop criteria to implementations of opinion dynamics on social networks.

References

1. F. Amblard and G. Deffuant. The role of network topology on extremism propagation with the relative agreement opinion dynamics. cond-mat/0404574, 2004.
2. E. Chattoe and N. Gilbert. Accepting government payment for new agri-environmental practices: A simulation approach to social complexity. Lund, Sweden, August 1999.
3. G. Deffuant, F. Amblard, G. Weisbuch, and T. Faure. How can extremism prevail? a study on the relative agreement interaction model. *Journal of Artificial Societies and Social Simulation (JASSS)*, 5(4), 2002. <http://jasss.soc.surrey.ac.uk/5/4/1.html>.
4. G. Deffuant, D. Neau, F. Amblard, and G. Weisbuch. Mixing beliefs among interacting agents. *Advances in Complex Systems*, 3:87–98, 2000.
5. S. Fortunato. The Krause-Hegselmann consensus model with discrete opinions. arXiv:cond-mat/0403670v1, march 2004.
6. J. R. P. French. A formal theory of social power. *Psychological Review*, 63(3):181–194, 1956.
7. S. Galam. Minority opinion spreading in random geometry. *The European Physical Journal B*, 25:403–406, 2002.
8. R. Hegselmann. Opinion dynamics: Insights by radically simplifying models. In D. Gillies, editor, *Laws and Models in Science*, pages 1–29. London, 2004.
9. R. Hegselmann and U. Krause. Opinion dynamics and bounded confidence models: Analysis, and simulation. *Journal of Artifical Societies and Social Simulation (JASSS)*, 5(3), 2002. <http://jasss.soc.surrey.ac.uk/5/3/2.html>.
10. R. Hegselmann and U. Krause. Collective dynamics of interacting agents when driven by PAM. Technical report, Modeling, Computing, and Mastering Complexity, 2003. preprint.
11. J. Lorenz. Mehrdimensionale Meinungsdynamik bei wechselndem Vertrauen. Master's thesis, University Bremen, Department of Mathematics, March 2003. <http://www-stuga.informatik.uni-bremen.de/mathematik/archiv/diplome/ jlorenz.zip>.
12. F. Schweitzer and J. A. Holyst. Modelling collective opinion formation by means of active brownian particles. *The European Physical Journal B*, 15:723–732, 2000.
13. D. Stauffer, A. Sousa, and C. Schulze. Discretized opinion dynamics of the Deffuant model on scale-free networks. *Journal of Artificial Societients and Social Simulation (JASSS)*, 7(3), 2004. <http://jasss.soc.surrey.ax.uk/7/3/7.html>.
14. G. Weisbuch, G. Deffuant, F. Amblard, and J. P. Nadal. Interacting agents and continuous opinion dynamics. working paper, Santa Fe Institute, 2001. <http://www.santafe.edu/ sfi/ publications/ wpabstract/ 200111072>.
15. G. Weisbuch, G. Deffuant, F. Amblard, and J. P. Nadal. Meet, discuss, and segregate! *Complexity*, 7(3), 2002.